Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Advanced Calculus

Subject Code : 4SC03MTC1 **Branch : B. Sc. (Mathematics)**

Semester : 3 Date : 3 / 12 / 2015 Time : 2:30 To 5:30 Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $z = x^2y + xy + 1$, find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$. **[2]**

b) Check whether the function $f(x,y) = \frac{x^2+y^2}{x+y}$ is homogeneous or not. **[2]**

c) Find $\beta(6, 11)$. **[2]**

d) If $u = x^2 + y^2, x = at^2, \ y = 2at$ then find $\frac{\partial u}{\partial t}$. **[2]**

e) If $x = r\cos\theta, \ y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$. **[2]**

f) Show that $f(x) = e^x$ is always increasing function for any value of x. **[2]**

g) Define: gamma function. **[2]**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Prove that $\lim_{(x,y)\to(0,0)} \frac{xy}{y^2-x^2}$ does not exist. **[5]**

b) If $x = r\cos\theta, \ y = r\sin\theta$ then prove that
$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$. **[5]**

c) If $u = (2x^2 + y^2)(\log x - \log y)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$. **[4]**

Q-3 **Attempt all questions** **(14)**

a) Prove that $y = x + 2$ is an asymptote of the curve $y = \frac{x^2+2x-1}{x}$. **[5]**

b) Find range of value of x for which $f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upward or downward. Also find point of inflection. **[5]**

c) Evaluate: $\int_0^2 x^4(8 - x^3)^{-\frac{1}{3}}dx$. **[4]**

Q-4 **Attempt all questions** **(14)**

a) State and prove Duplication formula. **[7]**

b) State and prove Taylor's series for function of two variables. **[7]**

Q-5		**Attempt all questions**	**(14)**
	a)	Find the maximum value of $x^2y^3z^4$ subject to $2x+3y+4z=a$ using Lagrange's method.	**[7]**
	b)	Find extreme values for $f(x,y)=x^3+3xy^2-3x^2-3y^2+4$.	**[7]**
Q-6		**Attempt all questions**	**(14)**
	a)	If u and v are functions of r and s and r,s are functions of x and y. Then prove that, $\frac{\partial(u,v)}{\partial(x,y)}=\frac{\partial(u,v)}{\partial(r,s)}\cdot\frac{\partial(r,s)}{\partial(x,y)}$.	**[5]**
	b)	If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x^2\frac{\partial^2u}{\partial x}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=2\cos 3u\sin u$.	**[5]**
	c)	If $x^3+3x^2y+6xy^2+y^3=1$, find $\frac{dy}{dx}$.	**[4]**
Q-7		**Attempt all questions**	**(14)**
	a)	State and prove Euler's theorem.	**[7]**
	b)	If $f(x)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$, prove that the partial derivatives exist at $(0,0)$. Also the function is not continuous at $(0,0)$.	**[7]**
Q-8		**Attempt all questions**	**(14)**
	a)	Use Taylor's series to expand $\sin x\cos y$ in power of $\left(x-\frac{\pi}{3}\right)\left(y-\frac{\pi}{4}\right)$.	**[5]**
	b)	Prove that $\Gamma(n)=2\int_0^\infty e^{-x^2}x^{2n-1}dx$.	**[5]**
	c)	Prove that $\beta(m,n)=\beta(n,m)$.	**[4]**